Stored Energy, Magnetic Forces and Dynamic Effects

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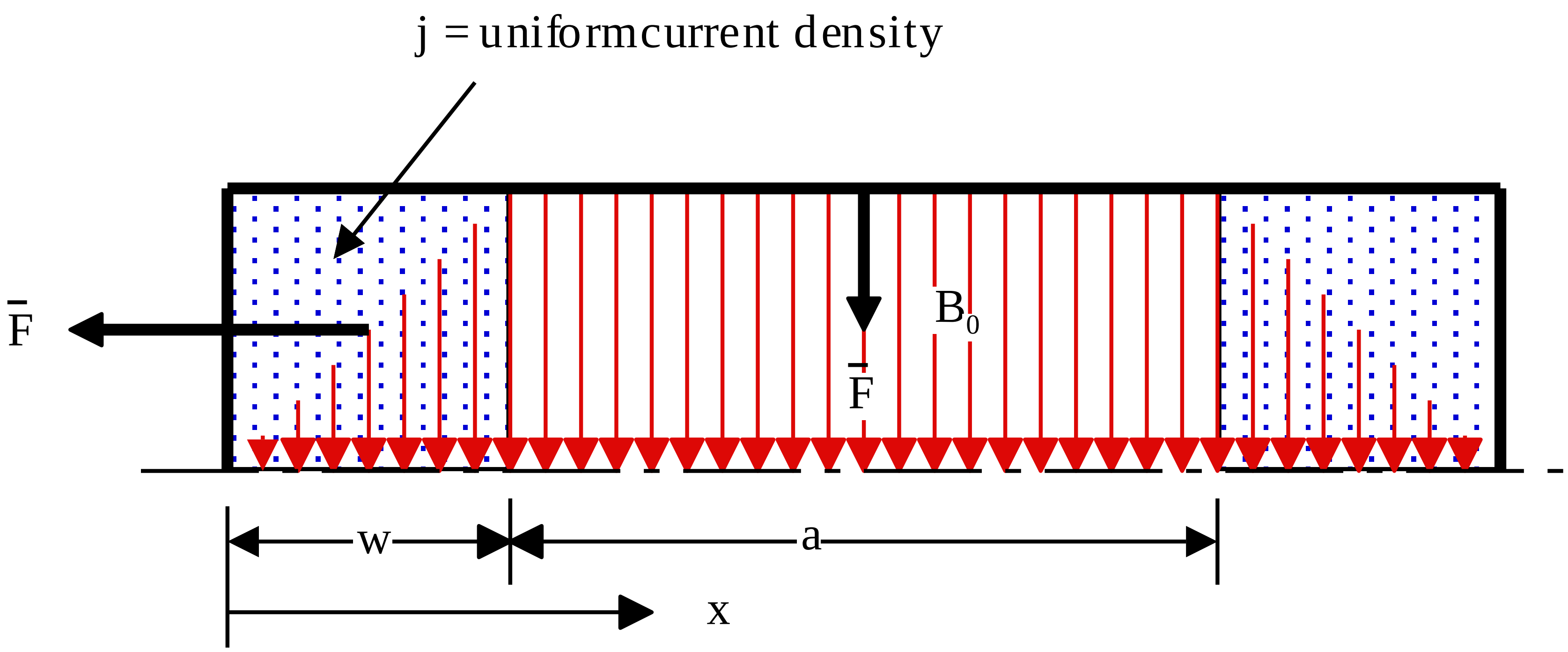
Outlook

- Magnetic Forces
- Stored Energy and Inductance
- Fringe Fields
- End Chamfering
- Eddy Currents

Magnetic Forces

- We use the figure to illustrate a simple example.



Magnetic Force on a Conductor

We use the expression, $\overline{F} = \int \overline{j} \times \overline{B}\, dv$

Integrated over the volume of the conductor.

$$NI = \frac{B_0 h}{\mu_0} \qquad j = \frac{NI}{wh} = \frac{B_0}{w\mu_0}$$

$$B = B_0 \frac{x}{w} \qquad dv = hLdx$$

L is the length *into the paper*.

$$\left|\overline{F}\right| = \int j\; B\, dv = \int \frac{B_0}{w\mu_0} B_0 \frac{x}{w} hLdx$$

$$= \frac{B_0^2 hL}{w^2 \mu_0} \int_0^w x dx = \frac{B_0^2 hL}{w^2 \mu_0} \frac{w^2}{2} = \frac{B_0^2 hL}{2\mu_0}$$

$$repulsive\ pressure = \frac{Force}{Area} = \frac{\left|\overline{F}\right|}{hL} = \frac{B_0^2}{2\mu_0}$$

Magnetic Force on a Pole

We use the expression, $\overline{F} = \int \rho \cdot \overline{H}\, dv$

where the magnetic charge density ρ is given by, $\rho = \frac{B_0}{h}$

The force is in the same direction as the *H* vector and is attractive.

$$H = \frac{B_0}{\mu_0} \frac{y}{h} \qquad dv = aLdy$$

$$\left|\overline{F}\right| = \int \rho H\, dv = \int \frac{B_0}{h} \frac{B_0}{\mu_0} \frac{y}{h} aLdy = \frac{B_0^2 aL}{h^2 \mu_0} \int_0^h ydy = \frac{B_0^2 aL}{h^2 \mu_0} \frac{h^2}{2} = \frac{B_0^2 aL}{2\mu_0}$$

$$attractive\ \ pressure = \frac{Force}{Area} = \frac{\left|\overline{F}\right|}{aL} = \frac{B_0^2}{2\mu_0}$$

Pressure

The *magnitudes* of the repulsive pressure for the current and the attractive pressure at the pole are *identical*. In general, the pressures *parallel* to the field lines are *attractive* and the forces *normal* to the field lines are *repulsive*. The pressure is proportional to the *flux density squared*.

$$pressure = \frac{B_0^2}{2\mu_0}$$

Example

Let us perform a calculation for a flux density of 5 kG = 0.5T.

$$pressure = \frac{B_0^2}{2\mu_0} = \frac{0.5^2}{2 \times 4\pi \times 10^{-7}} = 99{,}472\,\frac{Newtons}{m^2}$$

$$pressure_{\,@\,0.5\mathrm{T}} = 99{,}472\,\frac{Newton}{m^2} = 14.43\,\frac{lb_f}{in^2} \approx 1\,\mathrm{atmosphere}$$

Magnet Stored Energy

- The magnet stored energy is given by; $U = \frac{1}{2}\int HBdv$

the volume integral of the product of *H* and *B*.

Consider a *window frame* dipole field (illustrated earlier) with uniform field in the space between the coil. If we ignore the field in the coil and in the iron,

$$U = \frac{B_0^2}{2\mu_0} \times magnetic\ volume$$

Magnet Inductance and Ramping Voltage

Inductance is given by, $$L = \frac{2U}{I^2}$$

$$V = RI + L\frac{dI}{dt} = RI + \frac{2U}{I^2} \times \frac{I}{\Delta t} = RI + \frac{2U}{I\Delta t}$$

In fast ramped magnets, the resistive term is small.

$$V \approx \frac{2U}{I\Delta t} = \frac{2}{I\Delta t} \times \frac{B_0^2}{2\mu_0} \times volume \qquad I = \frac{B_0 h}{N\mu_0} \qquad volume = ahL$$

Substituting, $$V \approx \frac{1}{\frac{B_0 h}{N\mu_0}\Delta t} \times \frac{B_0^2}{\mu_0} \times ahL = \frac{B_0 NaL}{\Delta t}$$

Units and Design Options

The units are,

$$V \approx \frac{B_0 N a L}{\Delta t} = \frac{T m^2}{\sec} = \frac{Webers}{m^2} \times \frac{m^2}{\sec} = \frac{Volt - \sec}{\sec} = Volt$$

$$V \approx \frac{B_0 N a L}{\Delta t}$$

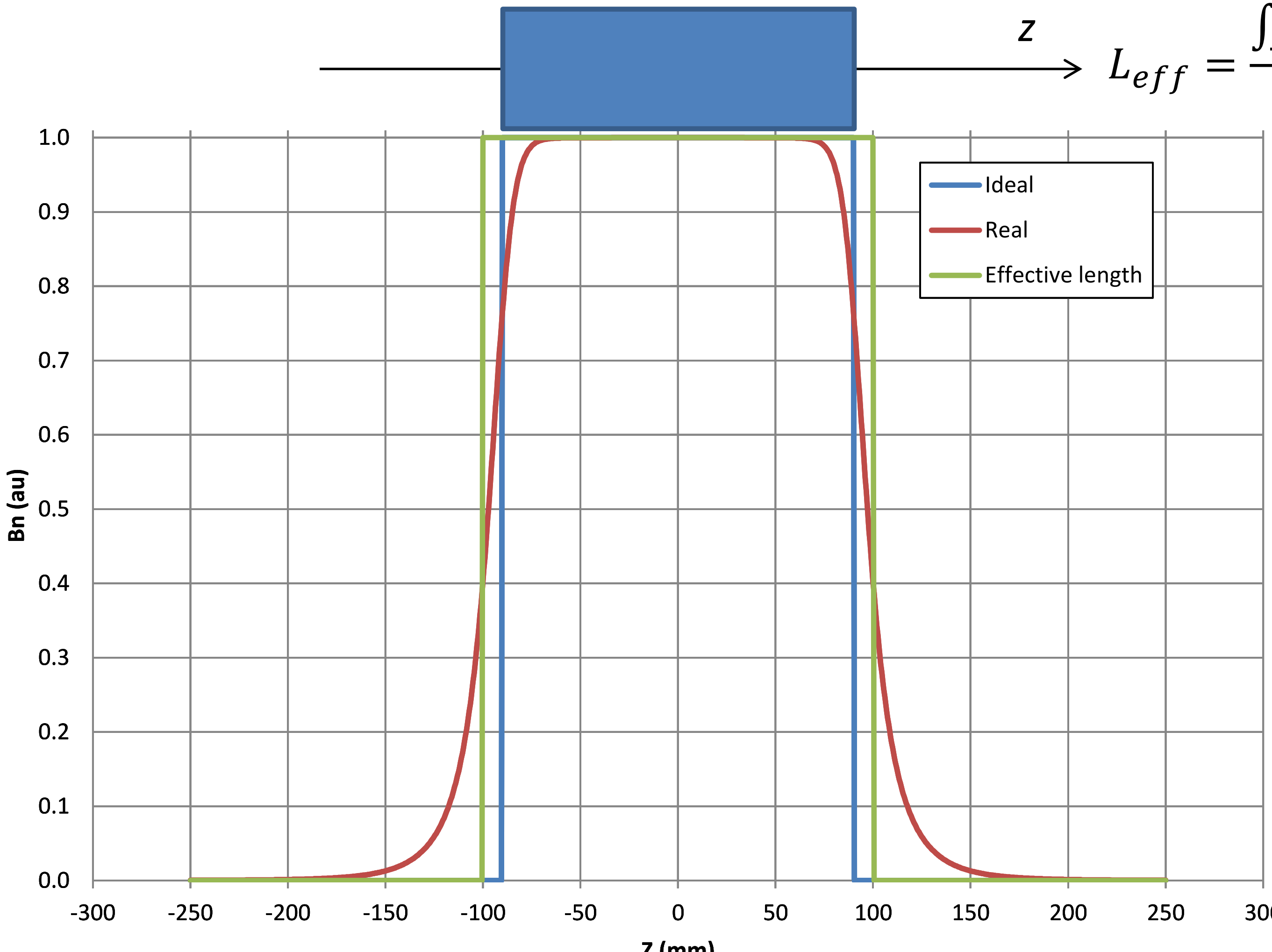
Given the field = B_0, pole width = *a*, Magnet Length = *L* and ramp time, Δ *t*, the only design option available for changing the voltage is the number of turns, *N*.

Other Magnet Geometries

Normally, the stored energy in other magnets (ie. H dipoles, quadrupoles and sextupoles) is not as easily computed. However, for more complex geometries, two dimensional magnetostatic codes will compute the stored energy per unit length of magnet.

Effective Length

$$L_{eff} = \frac{\int_{-\infty}^{+\infty} B_n(z)\, dz}{B_n(z=0)}$$



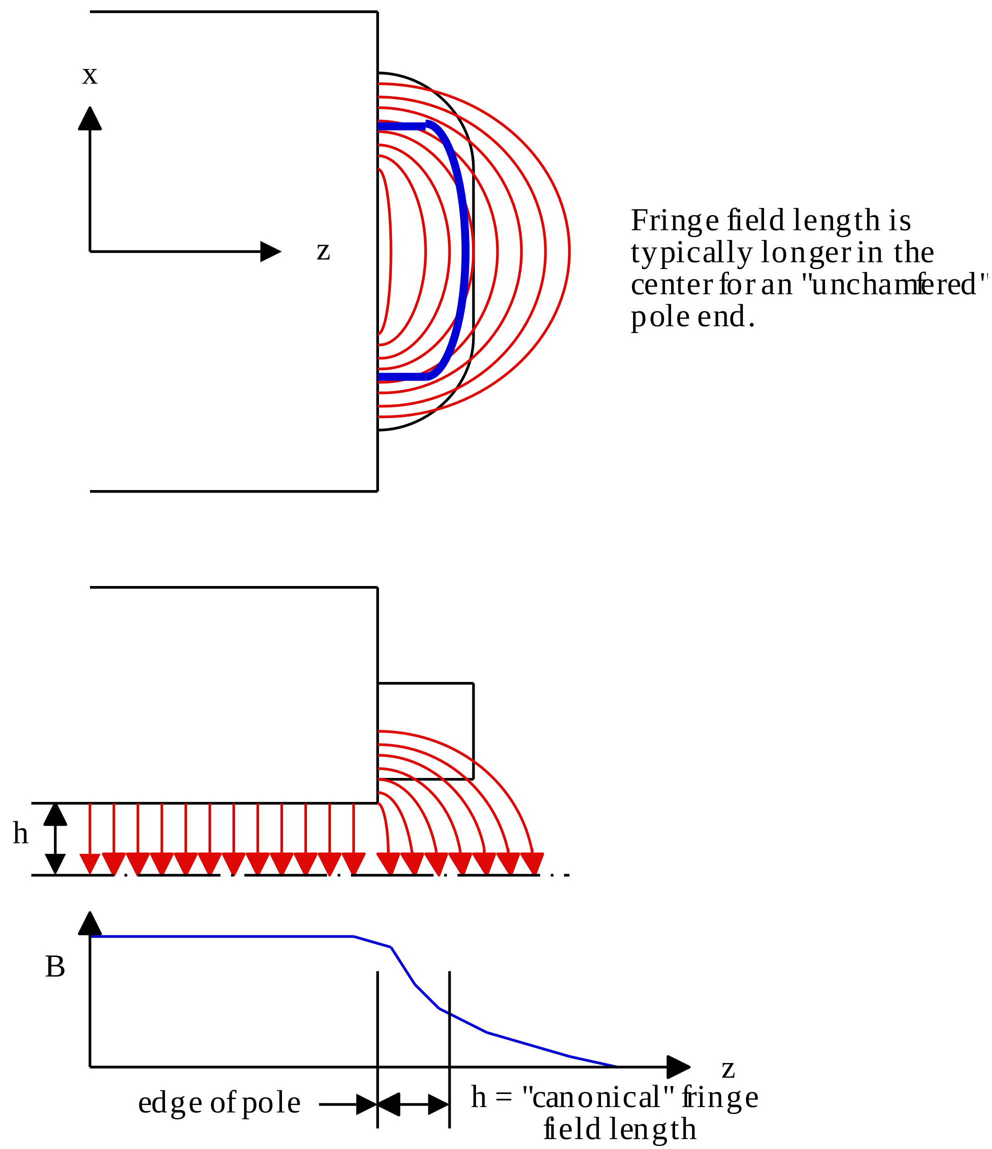
Fringe Fields and Effective Lengths

- Often, canonical rules of thumb are adopted in order to estimate the effective length of magnets.
  – Dipole fringe field length = 1 half gap at each end
  – Quadrupole fringe field length = 1/2 pole radius at each end.
  – Sextupole fringe field length = 1/3 pole radius at each end.

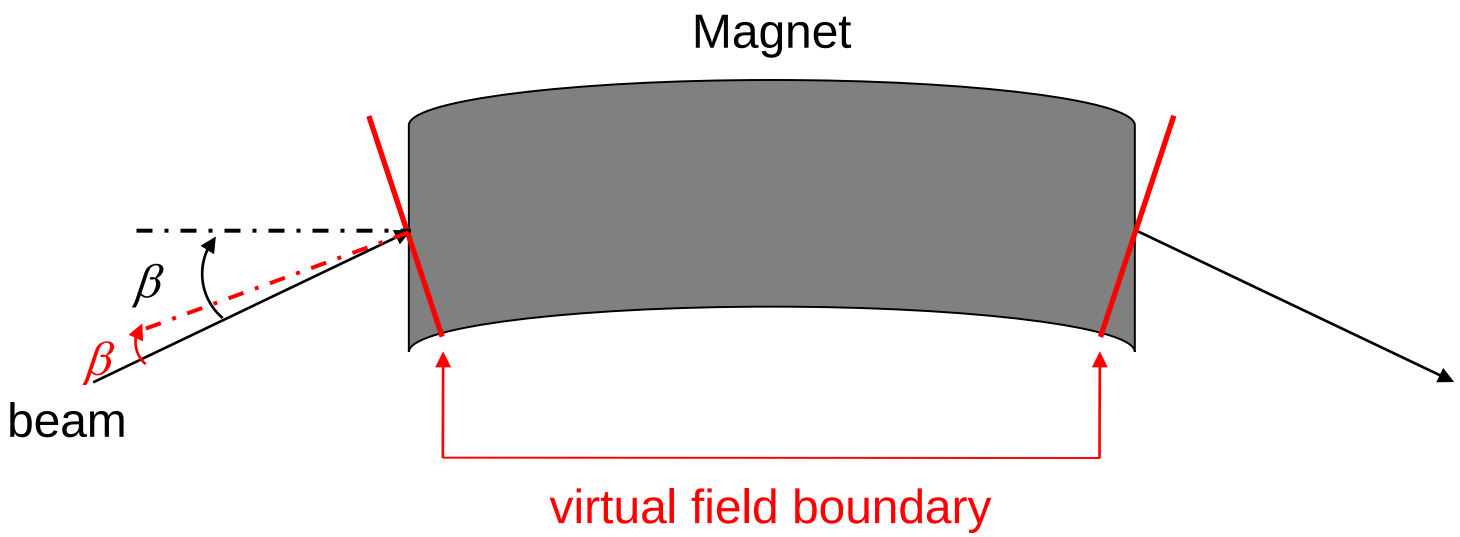
Three Dimensional Fringe Fields

The shape of the three dimensional fringe field contributes to the integrated multipole error of a magnet.

- Dipole Fringe Field
  - Typically, the fringe field is longer at the center of the magnet and drops off near the edges.
  - This distribution is approximately quadratic and the integrated multipole field looks like a sextupole field.



Virtual Field Boundary

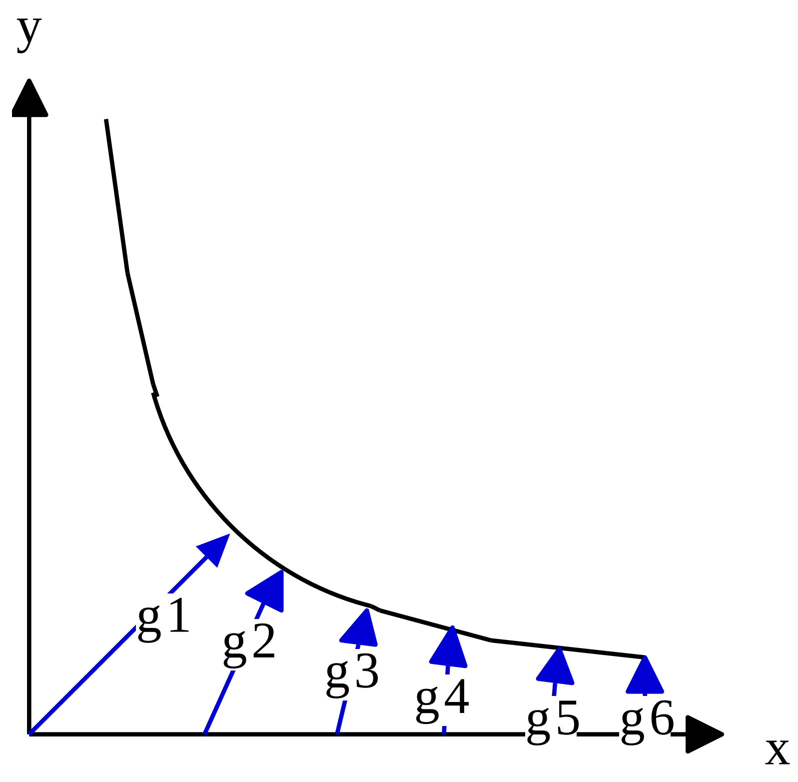


$$\frac{1}{Fx} = -\frac{\tan\beta}{\rho}$$

- The photograph shows a removable insert with a machined chamfer installed on the SPEAR3 prototype *gradient* magnet.
- The shape of the chamfer depth was determined empirically and was approximately parabolic. It was designed to reduce the integrated sextupole field.
- The chamfer shape was machined onto the end pack of subsequently manufactured production magnets.

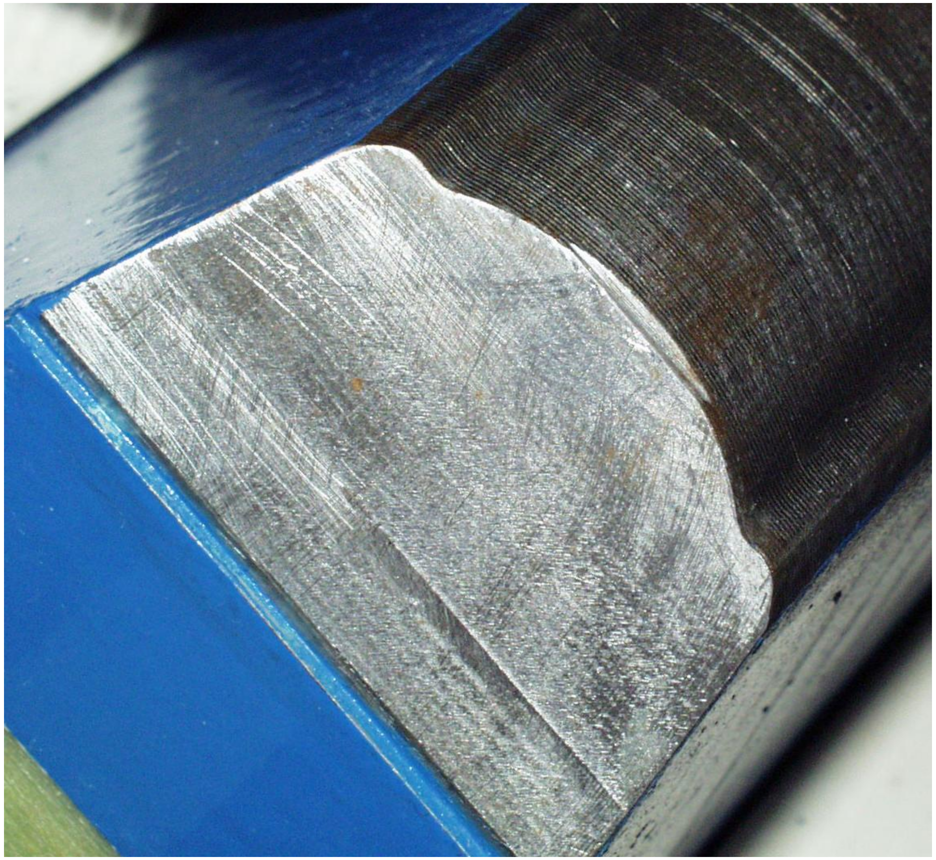
Quadrupole 3-Dimensional Fringe Field

- The quadrupole "gap" is largest at its center. The gap decreases as the distance from its center (g1>g2>g3>g4>g5>g6). Since the fringe field is roughly proportional to the magnet gap, it is longest near the magnet pole center.

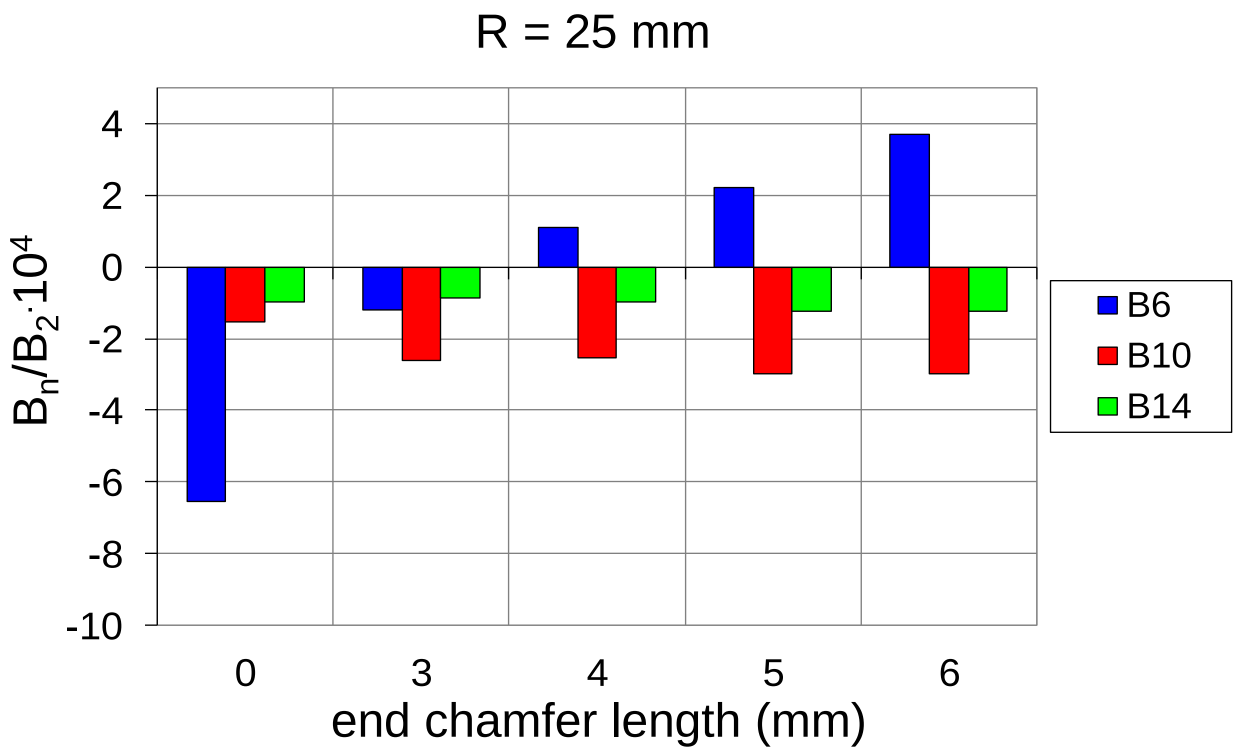


Quadrupole Chamfer

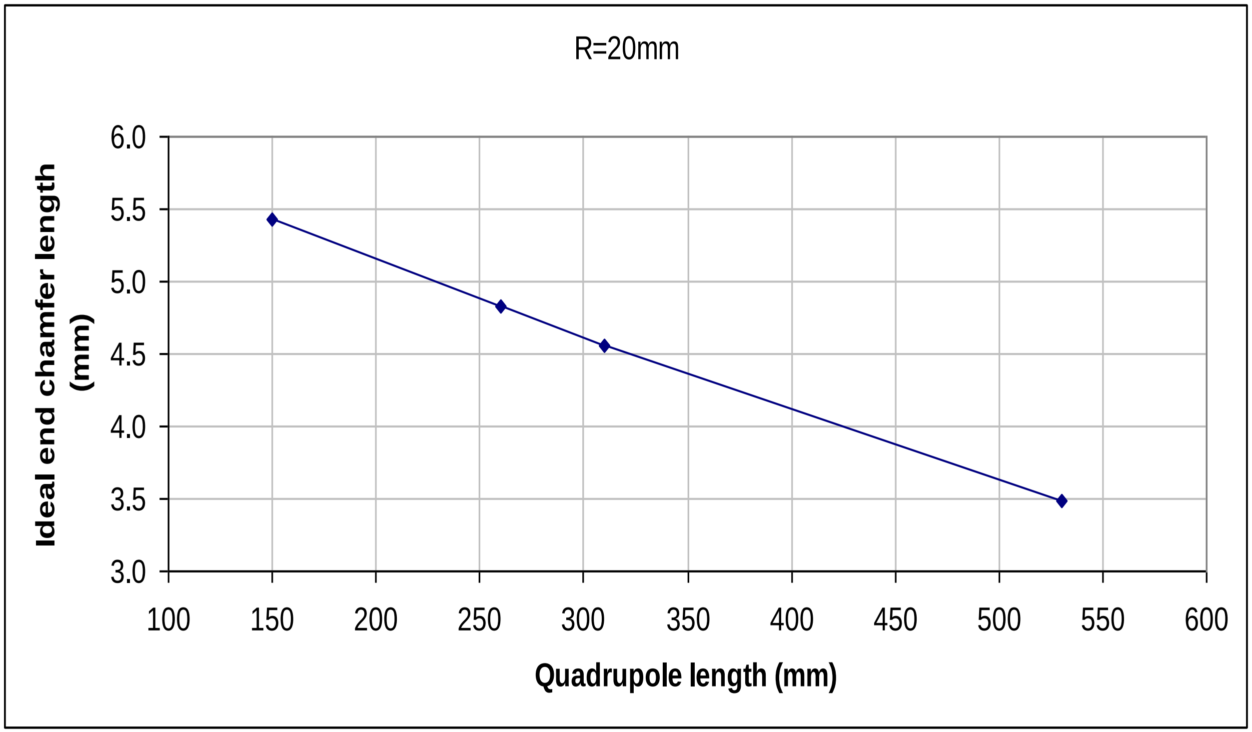
- The quadrupole pole chamfer is a straight angled cut, which shortens the pole at its center. The angle is cut such that the pole is longer near its edge.
- Again, this cut was determined empirically by trial an error, minimizing the n=6 integrated multipole error.



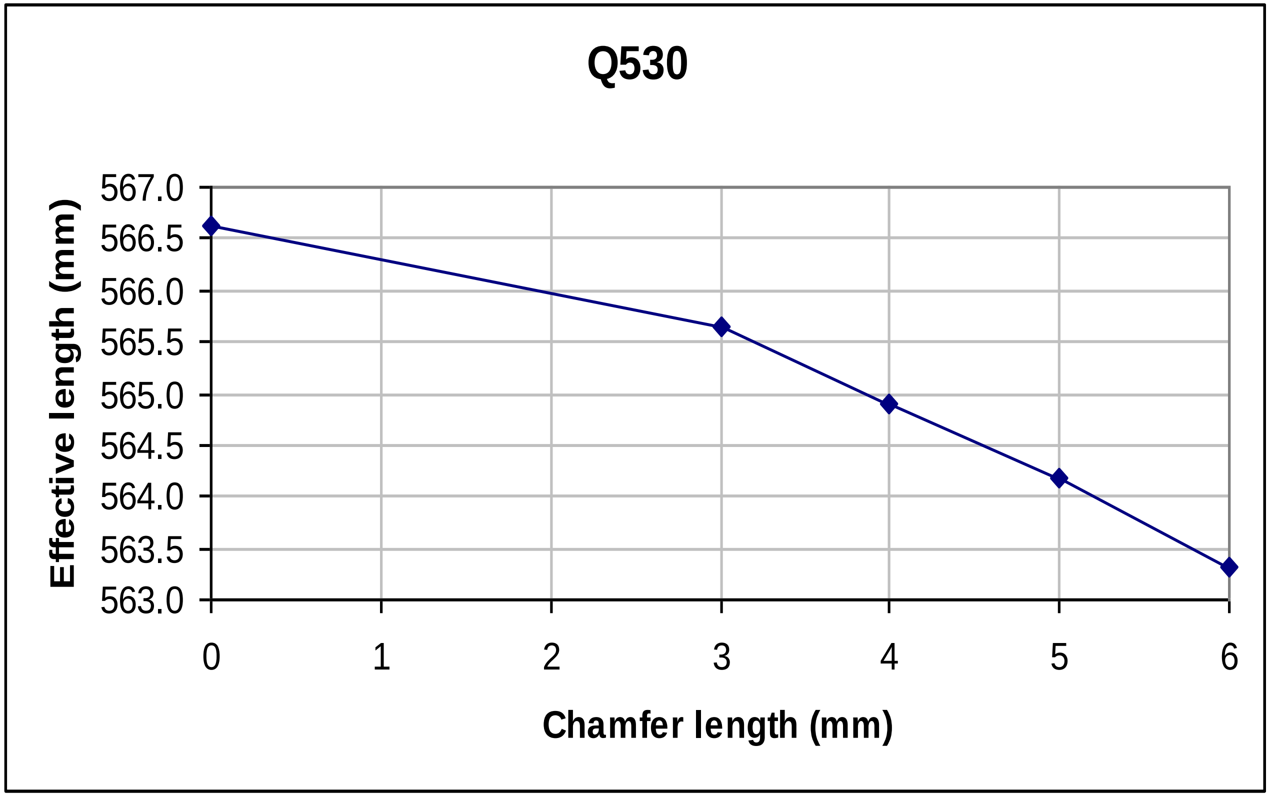
Harmonics as function of the end chamfer length



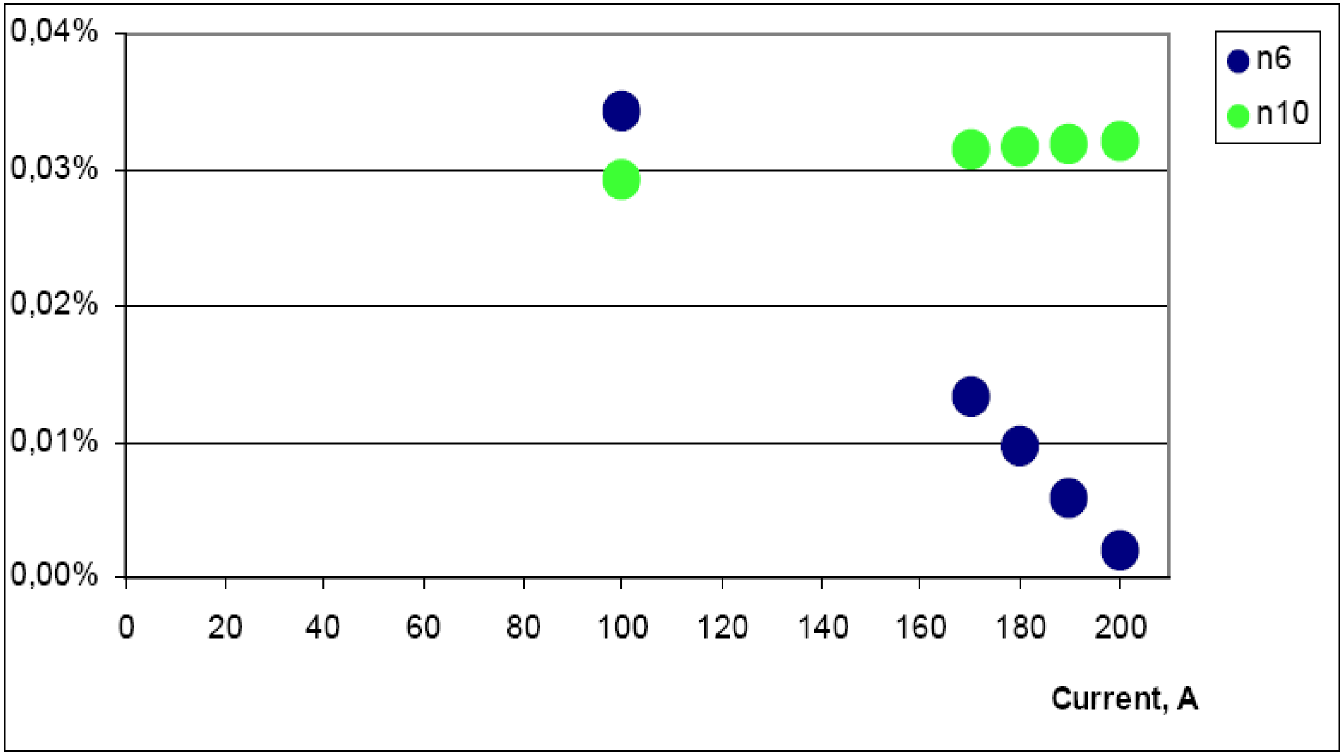
Ideal end chamfer length as function of quadrupole length



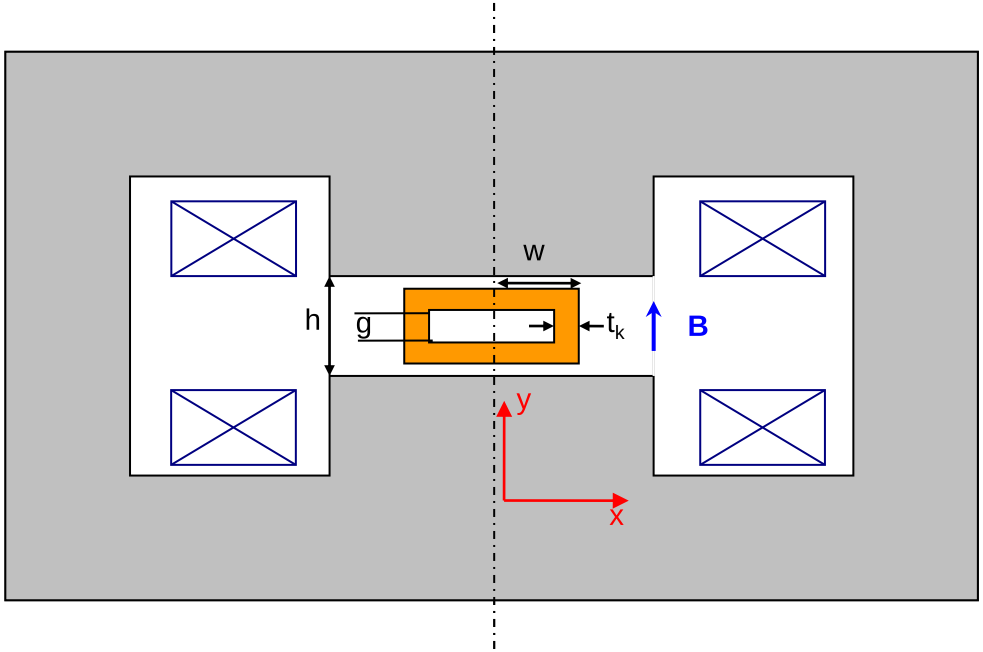
Effective length as function of the end chamfer length



Multipoles as function of the coil excitation



Dynamic effects



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{dt} \qquad \mathbf{B} = Bo.e^{i\omega t}.\hat{\mathbf{j}}$$

$$\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{\mathbf{j}} = -\dot{\mathbf{B}}\hat{\mathbf{j}}$$

$$\frac{\partial E_z}{\partial x} = i\omega B_o$$

$$Ez = \int i\omega B_o dx = i\omega B_o x$$

$$\mathbf{J} = \sigma \mathbf{E}$$

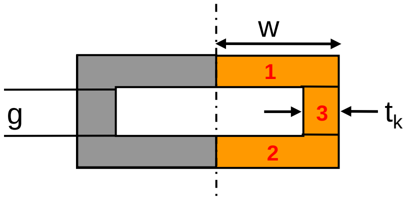
$$J_z = \sigma E_z = i\omega\sigma B_o x$$

Dynamic effects

$$I = \oint H.dl$$

$$I = \int \frac{B}{\mu_o}.dl + \int \frac{B}{\mu_{Fe}}.dl = \frac{gB_e}{\mu_o}$$

B_e stands for the magnetic field due to the eddy currents



$$I = I_1 + I_2 + I_3$$

$$I_1 = I_2$$

$$I = 2I_1 + I_3$$

$$I_i = \iint Jdxdy$$

$$I_1 = t_k \int_0^w i\omega\sigma B_o x dx = i\omega\sigma B_o t_k \frac{1}{2}\left(w^2\right)$$

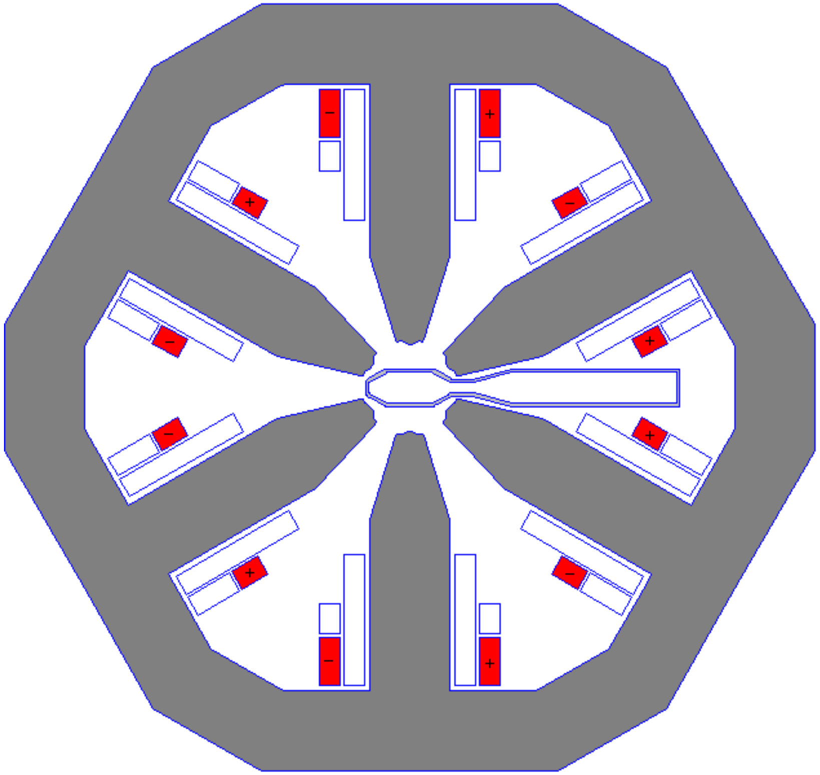
$$I_3 = g \int_{w-t_k}^{w} i\omega\sigma B_o x dx = i\omega\sigma B_o g \frac{1}{2}\left(wt - t_k{}^2\right)$$

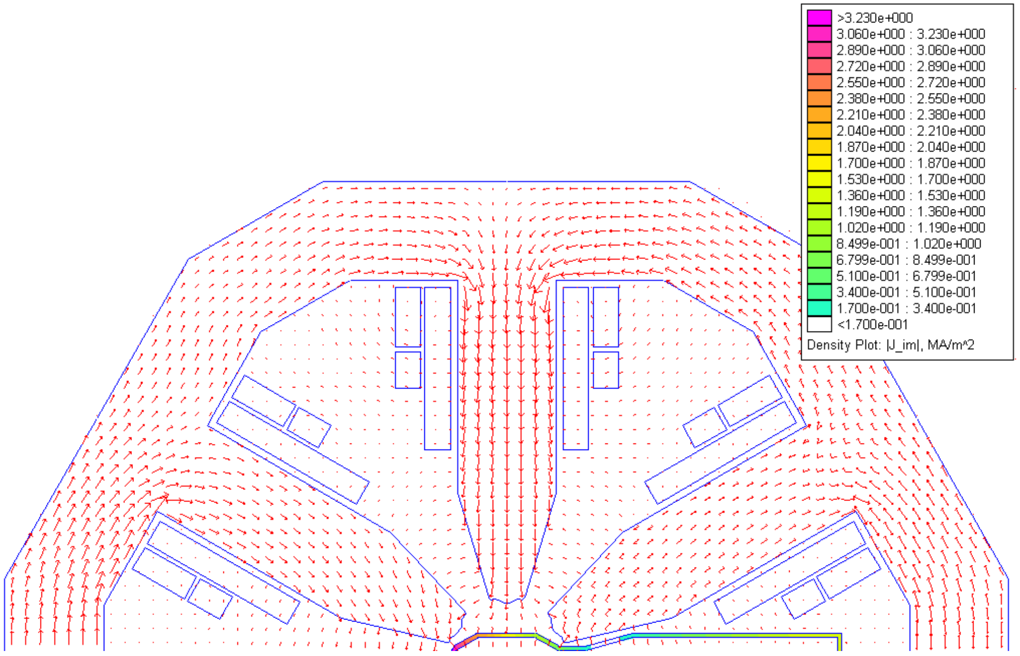
$$\boxed{B_e = i\frac{\omega\mu_o\sigma B_o t_k}{g}\left(w^2 + \frac{wg}{2} - \frac{t_k g}{2}\right)}$$

B_o(T)	0.145
frequency (Hz)	100
t_k (mm)	3
g (mm)	25
w (mm)	28
σ (MS/m)	1.334

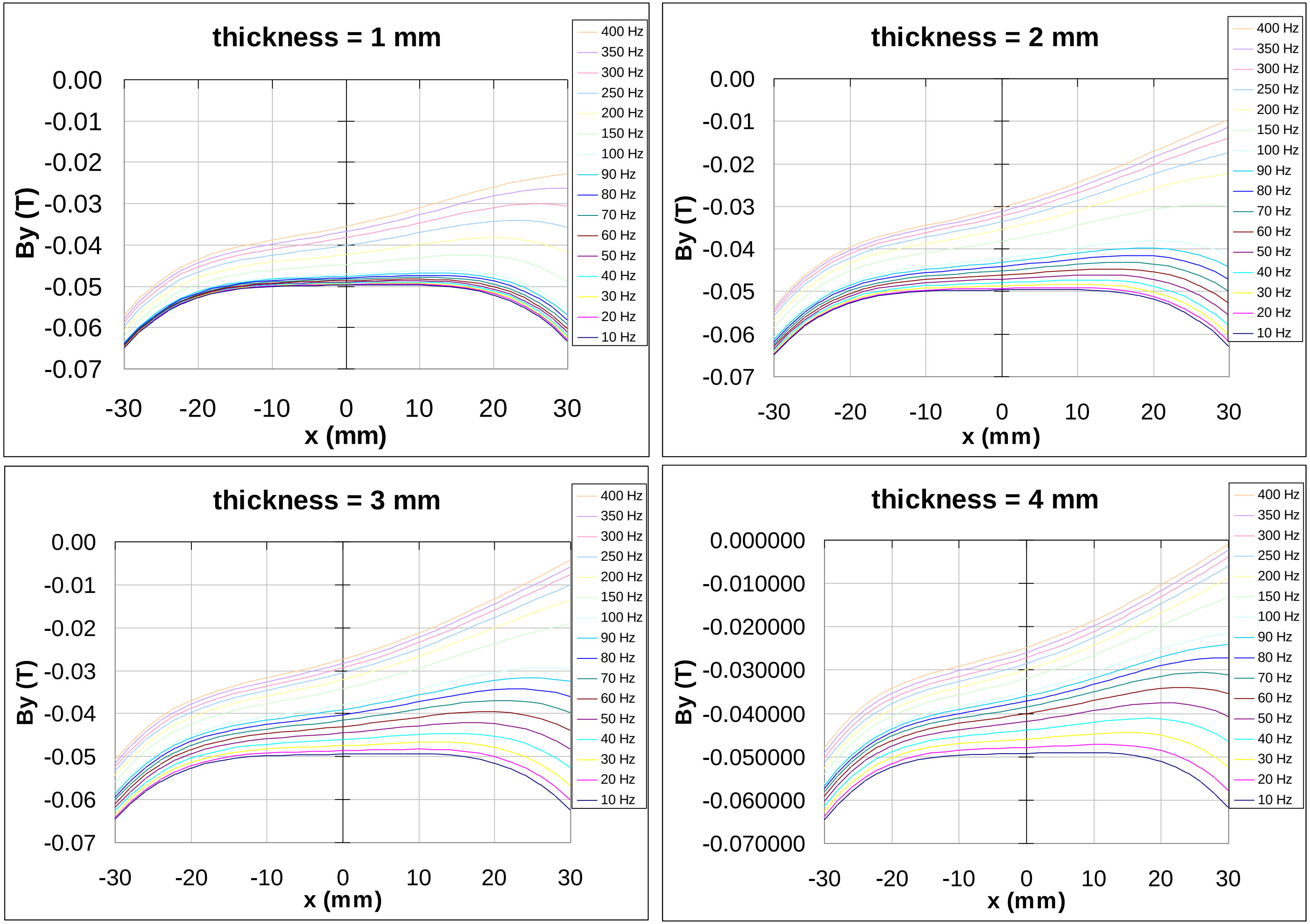
B_e (x=0) (theoretical)	0.0129	(T)
B_e (x=0) (FEMM)	0.0127	(T)

Real-case example



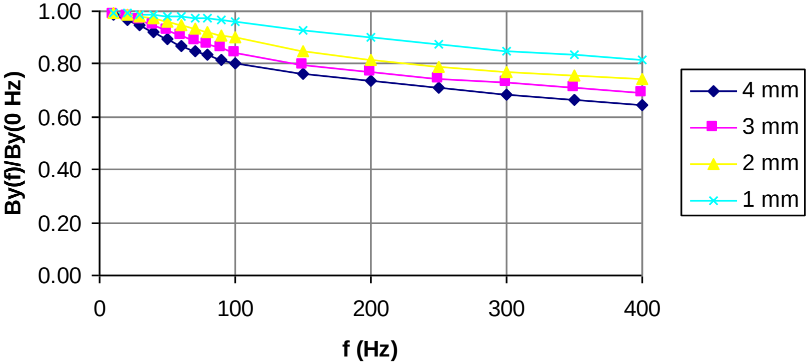


Real-case example

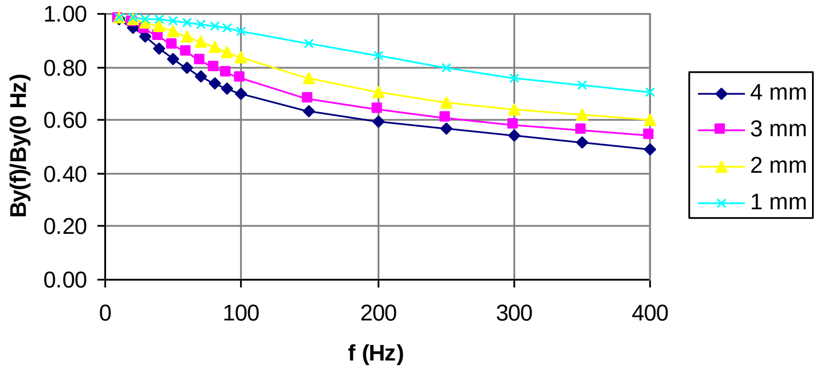


Real-case example

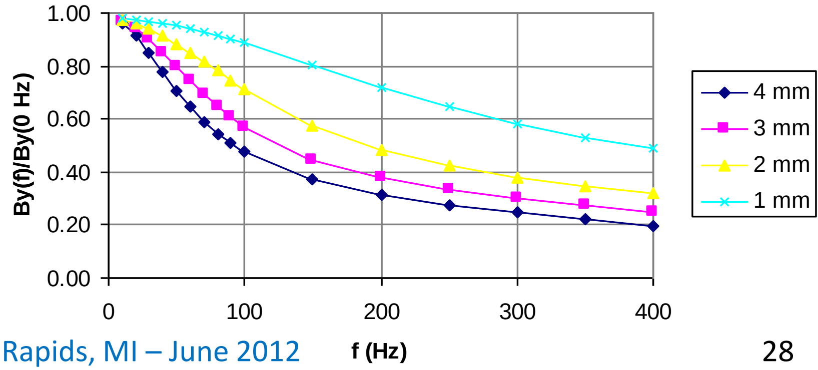
x = -20 mm



x = 0 mm



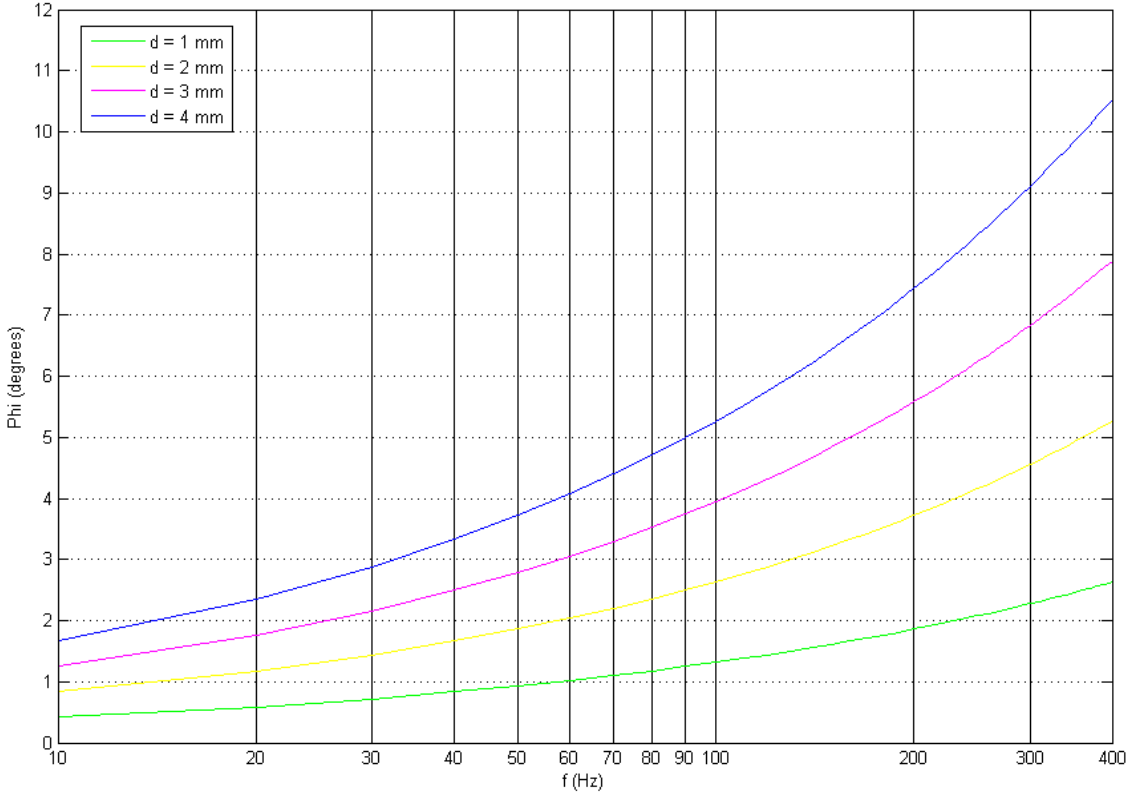
x = 20 mm



Phase delay

$$\phi = \frac{t_k}{\delta}\left[\text{radians}\right]$$

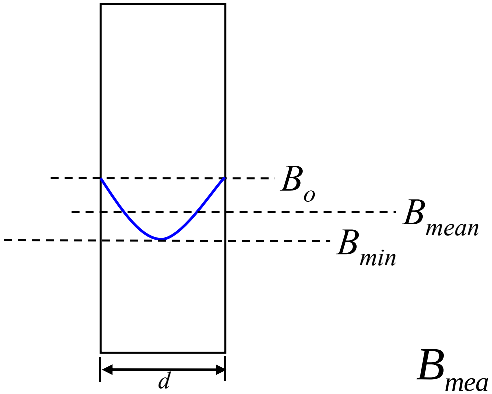
$$\delta = \sqrt{\frac{2}{\mu_r \mu_o \sigma \omega}}$$



Attenuation due to lamination on the material

$$\frac{\psi}{\psi_o} = \frac{2}{kd}\tan(kd/2)$$

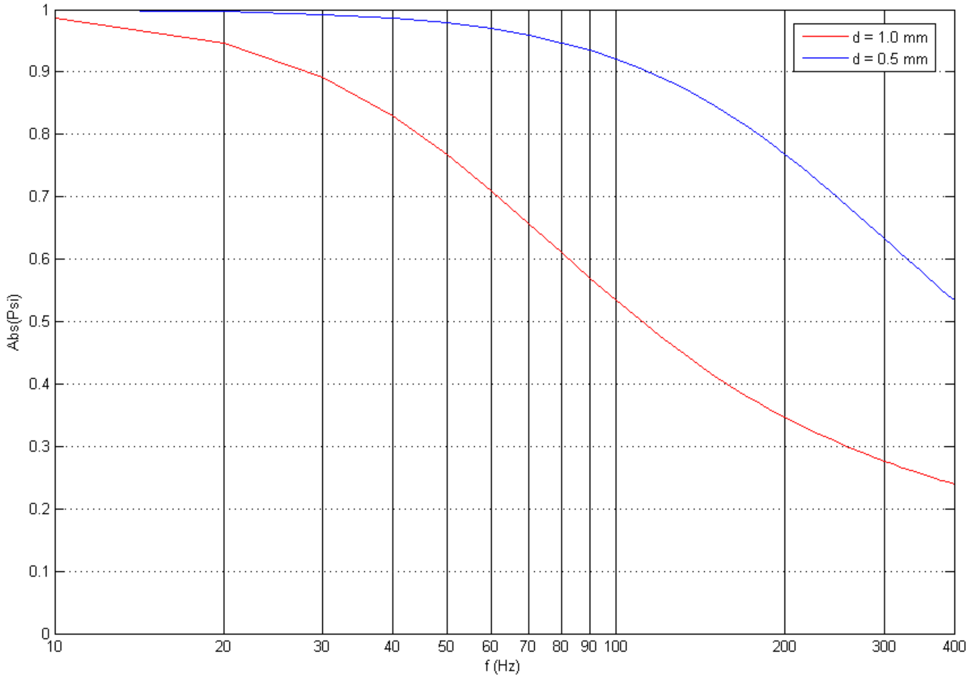
$$k^2 = -i\omega\mu\sigma$$



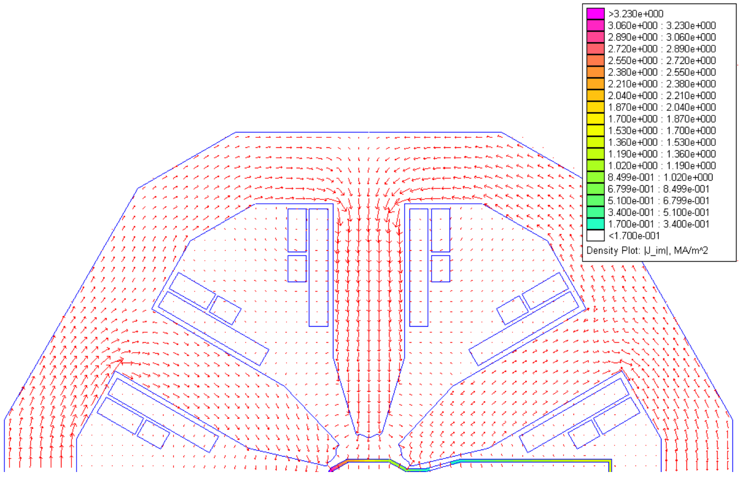
$$B_{mean} = \frac{B_{\min}}{\sqrt{2}}$$

Example

$\mu = 2200.\mu_o$
$\sigma = 9.93$ MS/m



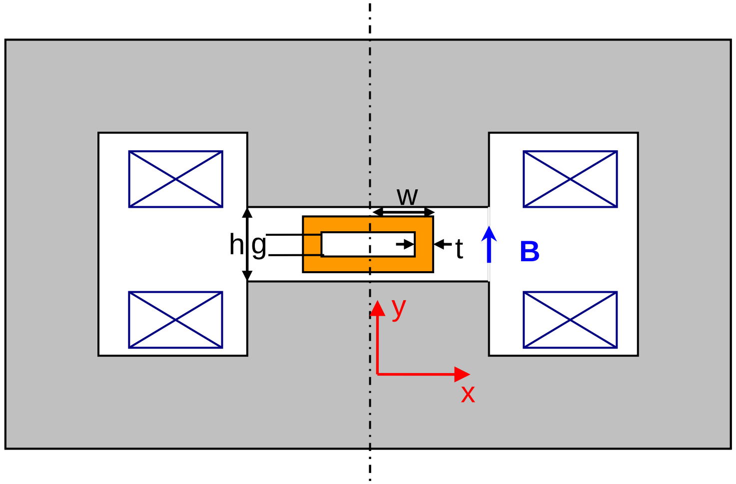
Real-case example summary



@400 Hz

Core lamination thickness \ Vacuum chamber thickness		2 mm	3 mm
		0.60	0.54
0.5 mm	0.67	0.40	0.36
1.0 mm	0.46	0.27	0.25

Induced sextupolar field due to eddy currents



$$J_z = \sigma E_z = \sigma \dot{B} x$$

$$I = \frac{Bh}{\mu_o}$$

$$I = t_k \sigma \dot{B} x^2$$

$$\boxed{B(x) = \mu_o \sigma \frac{t_k}{h} \frac{\partial B}{\partial t} x^2}$$

The sextupole strength (or moment) is defined by

$$k_2 = 2\mu_o \sigma F \frac{t_k}{h} \frac{1}{\rho B} \frac{\partial B}{\partial t}$$

$$F = 2\int_0^1 \left[x^2 + \left(\frac{g}{w}\right)^2 \left(1 - x^2\right)\right]^{1/2} dx$$

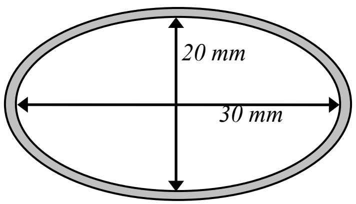
F=*1 when* g<<w, *and* F=*2 for a circular chamber*

$$B(t) = B_i + (B_e - B_i) f(t)$$

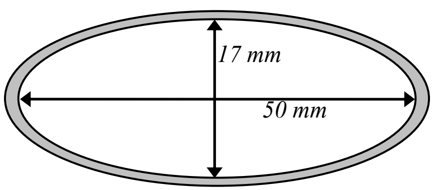
$$f(t) = \frac{1}{2}\left(1 - \cos(\omega t)\right)$$

Example – Alba booster dipole

ρ	11.4592	(m)
μ_o	$4\pi 10^{-7}$	(H/m)
σ	1.344	(MS/m)
Freq.	3	(Hz)
B_e	0.029	(T)
B_i	0.873	(T)

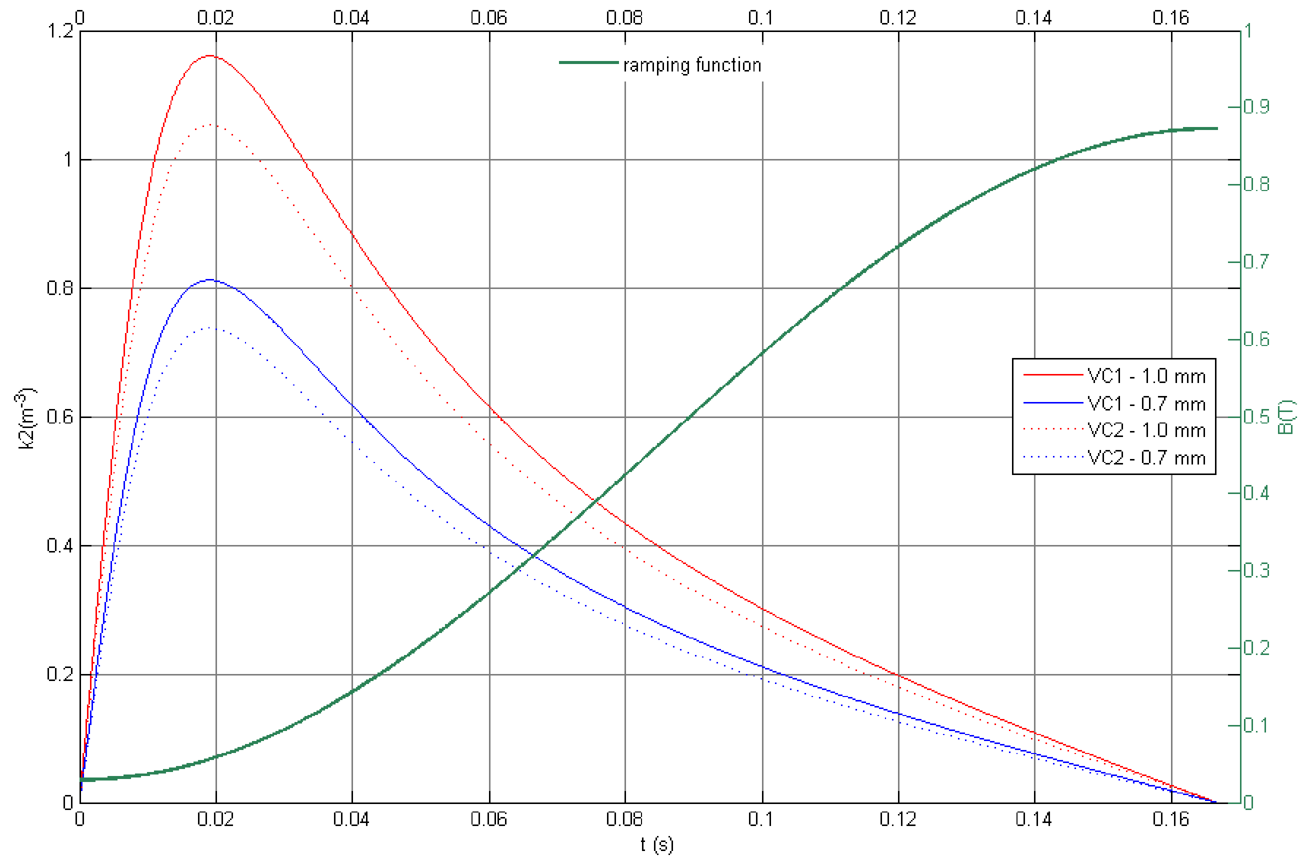


VC1



VC2

	F
VC1	1.5739
VC2	1.2141



Summary

This chapter showed a collection of “loose-ends” calculations:

- Magnetic Forces
- Stored Energy and Inductance
- Fringe Fields
- End Chamfering
- Eddy Currents

Those effects have an impact in the magnet design but also need to be taken into consideration into the magnet fabrication, power supply design, vacuum chamber design and beam optics.

Next…

Magnetic measurements